Factorisation of polynomials

Factor Theorem:

If p(x) is a polynomial of degree n≥1 and a is any real number, then

i) x-a is a factor of p(x),if p(a) =0,and

ii) p(a) =0,if x-a is a factor of p(x).

Exercise 2.4

1. Determine which of the following polynomials has (x+1) a factor:

i) x^3+x^2+x+1

ii)x^4+x^3+x^2+x+1

iii) $x^4 + 3x^3$+$3x^2$+x+1

iv) x^3- x^2- (2+$\sqrt{2}$) x+$\sqrt{2}$

Solution:

i) Let p(x) = x^3+x^2+x+1 and g(x) = (x - -1)

p(-1) = $(-1)^3$+$(-1)^2$+(-1)+1

=-1 +1-1+1 =0

So (x+1) is a factor of the given polynomial.

ii) Let p(x) = x^4+x^3+x^2+x+1 and g(x) =(x- -1)

p(-1) = $(-1)^4$+$(-1)^3$+$(-1)^2$+(-1) +1

= 1-1+1-1+1 =1

So (x+1) is not a factor of the given polynomial.

iii) Let p(x) = x^4+$3x^3$+$3x^2$+x+1 and g(x) = (x - -1)

p(-1) = $(-1)^4$+3$(-1)^3$+3$(-1)^2$+(-1) +1

= 1-3+3-1+1 =1

So (x+1) is not a factor of the given polynomial.

iv) Let p(x) = x^3-x^2- (2+$\sqrt{2}$)x +$\sqrt{2}$ and g(x) = (x- -1)

p(-1) = $(-1)^3$- $(-1)^2$-(2+$\sqrt{2}$) (-1) +$\sqrt{2}$

= -1-1+2+$\sqrt{2}$+$\sqrt{2}$ = 2$\sqrt{2}$

So (x+1) is not a factor of the given polynomial.

2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

i) p(x) = 2x^3+x^2-2x-1, g(x) = x+1

ii) p(x) = x^3+3x^2+3x+1, g(x) = x+2

iii) p(x) = x^3-4x^2+x+6, g(x) = x-3

<u>Solution:</u>

i) p(-1) = 2$(-1)^3$+$(-1)^2$-2(-1)-1

=-2+1+2-1=0

So g(x) is a factor of p(x).

ii) p(-2) = $(-2)^3$+3$(-2)^2$+3(-2)+1

=-8+12-6+1= -1

So g(x) is not a factor of p(x).

Iii) p(3) = 3^3-4(3^2)+3+6

= 27 – 36 +9 = 0

So g(x) is a factor of p(x).

3. Find the value of k, if x-1 is a factor of p(x) in each of the following cases

i)p(x) = x^2+x+k

ii) p(x) = 2x^2+kx+$\sqrt{2}$

iii)p(x) = kx^2- $\sqrt{2}$x+1

iv) p()x) = kx^2-3x+k

Solution:

i)If (x-1) is a factor of p(x),then p(1) will be zero.

$P(1) = 1^2+1+k=0$

$2+k=0$ then $k=-2$

ii) $p(1) = 2(1)^2+k(1) +\sqrt{2}=0$

$2+k+\sqrt{2}=0$ then $k = -2- \sqrt{2}$

iii) $p(1) = k(1)^2-\sqrt{2}(1)+1=0$

$k-\sqrt{2}+1=0$ then $k= \sqrt{2} -1$

iv) $p(1) = k(1)^2-3(1)+k = 0$

$k-3+k =0$ then $2k-3=0$

$k = \frac{3}{2}$

4. Factorise:

i) $12x^2-7x+1$

ii) $2x^2+7x+3$

iii) $6x^2+5x-6$

iv) $3x^2-x-4$

Solution:

i) The given polynomial is a quadratic polynomial of the form ax^2+bx+c

Consider ac = 12 x 1 = 12

The factors of 12 are 1,12,2,6,3,4.

We have to write b as the sum of two numbers whose product is ac.

So we can take the factors as 3 and 4.

So $12x^2-7x+1 = 12x^2-3x-4x+1$

$= 3x (4x-1)-(4x-1)$

$= (4x-1)(3x-1)$

ii) Consider ac = 2 x3=6

The factors of 6 are 1,6,2,3.

Among these we can take 1 and 6.

$2x^2+7x+3 = 2x^2+6x+x+3$

= 2x(x+3)+(x+3)

= (x+3)(2x+1)

iii) Consider ac= 6 x 6 =36

The factors of 36 are 1, 36,2,18,3,12,4,9,6.

Among these we can take 4 and 9 as factors.

Then $6x^2$+5x-6 = $6x^2$+9x-4x-6

= 3x(2x+3)-2(2x+3)

= (2x+3)(3x-2)

iv)Consider ac = 3 x 4 = 12

The factors of 12 are 1,12,2,6,3,4.

Among these we can take 3 and 4 as factors.

$3x^2$-x-4 = $3x^2$+3x-4x-4

= 3x(x+1)-4(x+1)

=(x+1)(3x-4)